CADI AYYAD MATHEMATICS CENTER

Inaugural Days

21-23 September 2022, Marrakesh

Ben Youssef Medrasa the oldest university in Marrakesh 1564-65

INVITED SPEAKERS

Mohamed Tahar Kadaoui Abbassi	FSDM Sidi Mohamed Ben Abdellah University of Fez, Morocco
Omar El Fallah	FSR Mohamed V University of Rabat, Morocco
Nourddine Igbida	FST University of Limoges, France
Zoubida Mghazli	FS Ibn Tofail University of Kenitra, Marocco
Nadia Raissi	FSR Mohamed V University of Rabat, Morocco
Abdelaziz Rhandi	University of Salerno, Italy
Hamidou Touré	Permanent Secretary of National Academy of Sciences of Burkina Faso
Vitaly Volpert	University of Lyon I, France
Enrique Zuazua	Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU), Germany

ABOUT

Cadi Ayyad University mathematics center, recently certified by the **CNRST** and gathering all the university's **mathematics and computer science** research structures, is organizing its inaugural days on **September 21-22 and 23, 2022.** The aim is to make the center known at the national level and bring together all its members and doctoral students around distinguished guest speakers. The days will in addition be an opportunity for Cadi Ayyad University's PhD students in mathematics to present their works.

Register via : http://mathcent.uca.ma/?page_id=893

Venue : Centre de conférence de l'Université Cadi Ayyad

Welcome !

In behalf of organization committee, We would like to welcome you to inaugural days of the **Cadi Ayyad Mathematics Center** to be held at Cadi Ayyad University of Marrakesh. Cadi Ayyad mathematics center, recently certified by the CNRST and gathering all the university's mathematics and computer science research structures, is organizing its inaugural days on September 21-22 and 23, 2022. The aim is to make the center known at the national level and bring together all its members and doctoral students around distinguished guest speakers. The days will in addition be an opportunity for Cadi Ayyad University's PhD students in mathematics to present their works.

Chair: Mohamed El Alaoui Talibi, FSSM Marrakesh.

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M'Hammed El Kahoui, FSSM Marrakesh.
El Hassan Essaky, FPS Safi.
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Said Boulite, ENSAM Marrakesh.
Said Gounane, EST Essaouira.
Fahd Karami, EST Essaouira.

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Part I

Abstracts

Plenary Lectures

Critical decay of singular values of Hankel operators on Bergman spaces

21 Sep
9h30-10h15

Omar El-Fallah
Mohammed V University in Rabat, Faculty of Sciences, LAMA
Rabat, Morocco

Let ω be a positive weight on the unit disc $\mathbb{D}$ and let A^2_ω be the Bergman space given by

$$A^2_\omega = \{f \in Hol(\mathbb{D}) : \ \|f\|_\omega = \left(\int_{\mathbb{D}} |f(z)|^2 \omega(z) dA(z)\right)^{1/2} < \infty\},$$

where dA is the normalized Lebesgue mesaure on $\mathbb{D}$. Let P_ω be the orthogonal projection from $L^2(\omega dA)$ onto A^2_ω. The Hankel operators $H_{\overline{\phi}}$ with the conjugate analytic symbol $\overline{\phi}$ is a densely defined operator from A^2_ω into $L^2(\omega) \ominus A^2_\omega$ given by

$$H_{\overline{\phi}} f = \overline{\phi} f - P_\omega(\overline{\phi} f),$$

Complete descriptions of bounded and compact Hankel operators were obtained by several authors and for a large class of weights. In this talk we are interested in the behavior of the singular values of compact Hankel operators. First, we give estimates of trace of $h(|H_{\overline{\phi}}|)$ for any convex function h, where $|H_{\overline{\phi}}| = \left(H^*_{\overline{\phi}} H_{\overline{\phi}}\right)^{1/2}$. Then we give some estimates of the singular values, $s_n(H_{\overline{\phi}})$, of $H_{\overline{\phi}}$.

For radial weights, we prove that the critical decay of $(s_n(H_{\overline{\phi}}))_n$ is achieved by $(s_n(H_{\overline{z}}))_n$. Then we characterize symbols for which $s_n(H_{\overline{\phi}}) = O(s_n(H_{\overline{z}}))$. The exact asymptotic behavior of $(s_n(H_{\overline{\phi}}))_n$ will be given in this critical case.
(This is a joint work with Marouane Bourass, Ibrahim Marrhich and Hatim Naqos)

Congestion pedestrian flow and constrained evolution PDEs

21 Sep.
10h45-11h30

Noureddine IGBIDA

University of Limoges, France

The purpose of the presentation is to show how to introduce new strategies for the correction phase in the Prediction-Correction models for congestion crowd motion. We will focus the conference on two types of strategy and we will present the main theoretical and numerical results that we obtain on these models.

Mathematical modelling of respiratory viral infections

21 Sep.
14h30-15h15

Vitaly Volpert

Institut Camille Jordan, UMR 5208 CNRS University Lyon 1, France

In this lecture we will present an overview of recent works on mathematical modelling of respiratory viral infections at the individual and population levels. We will begin with the investigation of infection progression in cell cultures and in tissues of human body. We will determine viral load and infection spreading speed and we will apply these results to evaluate infectivity and severity of symptoms for different variants of the SARS-CoV-2 infection. Next, using the estimates of the infection transmission rate, we will present new immuno-epidemiological models and will use them to evaluate the epidemiological situation for the ongoing COVID-19 pandemic.

Control and Machine Learning

22 Sep.
9h00-9h45

Enrique ZUAZUA
Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU), Germany

In this lecture we shall present some recent results on the interplay between control and Machine Learning, and more precisely, Supervised Learning and Universal Approximation.

We adopt the perspective of the simultaneous or ensemble control of systems of Residual Neural Networks (ResNets). Roughly, each item to be classified corresponds to a different initial datum for the Cauchy problem of the ResNets, leading to an ensemble of solutions to be driven to the corresponding targets, associated to the labels, by means of the same control.

We present a genuinely nonlinear and constructive method, allowing to show that such an ambitious goal can be achieved, estimating the complexity of the control strategies.

This property is rarely fulfilled by the classical dynamical systems in Mechanics and the very nonlinear nature of the activation function governing the ResNet dynamics plays a determinant role. It allows deforming half of the phase space while the other half remains invariant, a property that classical models in mechanics do not fulfill.

The turnpike property is also analyzed in this context, showing that a suitable choice of the cost functional used to train the ResNet leads to more stable and robust dynamics.

This lecture is inspired in joint work, among others, with Borjan Geshkovski (MIT), Carlos Esteve (Cambridge), Domènec Ruiz-Balet (IC, London) and Dario Pighin (Sherpa.ai).

Sur la géométrie Riemannienne du fibré tangent

22 Sep.
9h45-10h30

Mohamed Tahar Kadaoui Abbassi
Sidi Mohamed Ben Abdellah University of Fez, Morocco

Dans de cette conférence, on va présenter les étapes clés du développement de la recherche en géométrie (pseudo-)Riemannienne du fibré tangent à une variété Riemannienne. On va aussi commenter quelques questions d'actualité dans ce domaine de recherche.

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22 Sep.
14h30-15h15

Modeling some biological phenomena via the porous media approach

Zoubida Mghazli
Ibn Tofail University of Kenitra, Marocco

Many biological systems can be modeled by the "porous medium approach", such as the diffusion of nutrients and other macromolecules through and in biological tissues. In this presentation, after a brief introduction of the "porous medium approach", we present some biological systems viewed through this approach. This will mainly concern the process of biodegradation of household waste and the flow of water in the plant root.

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23 Sep.
9h00-9h45

Modélisation mathématique du développement durable, accent sur la stabilité, la stabilisabilité, la viabilité des systèmes complexes en jeu

Nadia RAISSI & Mustapha SERHANI
Mohamed V University of Rabat, University Moulay Ismail of Meknes, Morocco

L'expression "développement durable", est entrée dans l'usage populaire. Elle fait allusion à un souci de conservation pour les bénéfices à long terme de l'humanité. Une préoccupation d'autant plus pertinente au vu du chaos provoqué par les changements climatiques et de la conjoncture socio - économique au Maroc. Force est de constater, que lancement des programmes de restructuration des politiques de développement, accentue la pression sur les besoins en ressources, dans un contexte aggravé par le stress hydrique et la crise énergétique. Le secteur de la pêche notamment, illustre à lui seul les défis et opportunités du programme de l'économie bleue. Seule une gestion responsable des ressources halieutiques, pourrait rallier deux objectifs a priori contradictoires, développement économique et conservation des stocks. Ce double objectif peut être formulé comme la recherche d'un équilibre écosystémique stable. Incidemment, la modélisation mathématique permet de décrire les mécanismes complexes de l'activité. De plus, l'analyse des modèles élaborés, permet de rationaliser et d'améliorer les décisions pour une gestion durable et responsable, de ce secteur . Par ailleurs, il est urgent de mettre en œuvre des solutions écologiques

de traitement des eaux. A travers une modélisation mathématique appropriée, les principaux processus de traitement peuvent être décrits et une analyse de ces modèles permet d'en évaluer l'efficacité [3]. Face à la rareté des énergies fossiles, le Maroc s'est placé en fer de lance dans la recherche de solutions en énergie renouvelable. Ainsi, des programmes de transformation des déchets ménagers en énergie sont à l'étude au niveau de plusieurs communes au Maroc. Nous avons élaboré et analysé un modèle [2] et son application à la gestion d'une décharge publique dans les environs de Rabat est en cours de réalisation. Sur un tout autre registre, tout en appliquant la même méthodologie, nous avons développé des modèles d'évolution des tumeurs et leur thérapies ([1],[4]). Cette approche commune à ces différentes applications, s'appuie sur des outils mathématiques de plus en plus sophistiqués. Les concepts d'optimisation dynamique développés et utilisés sont essentiellement, la stabilité des systèmes d'évolution en jeu, leur stabilisabilité, et leur viabilité. Les succès et les limites de cette approche est examinée à travers leur modélisation tout en interrogeant leurs pertinences. Dans un souci de pragmatisme, les questions suivantes méritent notre attention : L'intervention d'une autorité de gestion suffit-elle à améliorer les performances en matière de durabilité? Un comportement coopératif de toutes les parties prenantes pourrait-il donner de meilleurs résultats? L'introduction du développement récent de la "théorie d'Hamilton Jacobi Bellmann" à l'analyse non lisse dans la modélisation de cette question s'avère une bonne piste de recherche ([1], [5]).

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23 Sep.
9h45-10h30

From dynamical systems to stochastic and partial differential equations

Abdelaziz Rhandi
University of Salerno, Italy

In mathematical modelling it is known that many natural phenomena can be modelled by ordinary differential equations of the following type:

$$(ODE) \quad \begin{cases} dX(t,x) = F(X(t,x))dt, \quad t \geq 0, \\ X(0,x) = x \in \mathbb{R}^N, \end{cases}$$

where $F : \mathbb{R}^N \to \mathbb{R}^N$ a given function. If a solution exists and unique, then it satisfies

$$X(t+s,x) = X(t, X(s,x)), X(0,x) = x \in \mathbb{R}^N, t, s \geq 0.$$

Hence the trajectories

$$S_t : \mathbb{R}^N \to \mathbb{R}^N; S_t x := X(t,x), t \geq 0$$

describe a dynamical system. Comparing such trajectories and those given by experiments, one sees that in most cases they differ from each other. This is caused by some stochastic noise and leads to the following modification, which seems more reasonable

$$(SDE) \begin{cases} dX(t,x) = F(X(t,x))dt + Q(X(t,x))dB(t), t > 0, \\ X(0,x) = x \in \mathbb{R}^N, \end{cases}$$

where $B(t)$ is a standard Brownian motion and $Q(x) \in \mathcal{L}\left(\mathbb{R}^N\right)$. The Itô Formula permits us to obtain the relationship between the above stochastic differential equation and the following parabolic PDE, called also Fokker-Planck equation. Given $f \in C_b\left(\mathbb{R}^N\right)$, the function

$$T(t)f(x) := \mathbb{E}(f(X(t,x))) \left(= \int_{\mathbb{R}^N} f(y)p(t,x,dy)\right)$$

solves the Fokker-Planck equation

$$(FP)\begin{cases} \frac{\partial u}{\partial t}(t,x)=\sum_{i,j=1}^{N} a_{ij}(x)D_{ij}u(t,x)+F(x)\cdot\nabla u(t,x), \quad t>0, \\ u(0,x)=f(x), \quad x\in\mathbb{R}^N \end{cases}$$

$(a_{ij}(x)) := \frac{1}{2}Q(x)Q(x)^*$. One has to note that in general the solution of (FP) is not always unique. The above given solution is the minimal one among all positive solutions that can be obtained for $f\geq 0$.

In this talk we recall briefly some important results for dynamical systems, explain how to solve (FP), how to obtain uniqueness and how to estimate the transition probabilities p for a large class of Fokker-Planck equations.

Existence and Controllability Results in the $\alpha-$norm and Kneser Property for Nonlinear Differential Inclusions in Banach Space

23 Sep.
15h-15h45

Hamidou TOURÉ
Permanent Secretary of National Academy of Sciences of Burkina Faso

In this work, we study the existence of mild and extremal solutions of first-order impulsive semilinear functional differential inclusions with finite delay in a separable Banach space X. We consider the following class of semilinear impulsive differential inclasions

$$\begin{cases} \frac{d}{dt}\left[u(t)-H\left(t,u_t\right)\right]+Au(r)\in f\left(r,u_k\right) \text{ for } t\in J=[0,b], r\neq r_k \\ \Delta u|_{t-t_k}\in l_k\left(u\left(t_k^-\right)\right) \text{ for } k=1,\ldots,m \\ u_0=\varphi\in C_\alpha \end{cases}$$

where $-A: D(A)\to X$ is the infinitesimal generator of a compact analytic semigroup of uniformly bounded linear operators, for $0<\alpha<1; A^\alpha$ is the fractional α-power of A, C_a is defined by

$$C_\alpha=\left\{\begin{array}{l} \psi\in C;\psi(\theta)\in D\left(A^a\right)\text{for}\theta\in[-\Gamma,0]<0,\ A^a\psi\in C, \\ \psi \text{ continuous everywhere except for a finite number of points} \\ \text{ at which}\psi\left(s^-\right) \text{ and } \psi\left(s^+\right) \text{ exist and } \psi\left(s^-\right)\neq\psi\left(s^+\right) \end{array}\right\}.$$

For $u_f\in C\left([-r,b],D\left(A^a\right)\right], b>0$, and $t\in[0,b]$ the history function $u_t\in C_\alpha$ is defined by

$$u_f(\theta)=u(t+\theta) \text{ for } \theta\in[-r,0]$$

$f : J \times C \to \mathsf{P}(X)$ and $H : J \times C_\alpha \to \mathsf{P}(X_\alpha)$ are a closed, bounded and convex valued multivalued maps. $I_k : X \to \mathsf{P}(X)$ are multivalued maps with closed, bounded and convex values, $u(t_k^-)$ and $u(t_k^+)$ represent the left and right limits at $t = t_k$ of $u(t)$ and $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = b$, and $\mathsf{P}(X)$ denotes the family of nonempty subsets of X. Impulsive differential equations have become more important in recent years in some mathematical models of real process and phenomena studied in physics, chemical tech- nology, population dynamics biotechnology and economics. In the first part, we prove existence of mild and extremal mild solutions for first-order semilinear non densely defined impulsive functional differential inclusions in a separable Banach space with local and nonlocal conditions. Secondly, we study the controllabillity of the system governed by the semilinear functional differential inclusion.

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tional differential equations with unbounded delay, Nonlinear Analysis, TMA 28, (3), 513-531, (1997).

Part II

Abstracts

Short Communications

Session 1

A novel model based on time-fractional derivative for noise removal

21 Sep.
11h30-11h50

Anouar Ben Loghfyry

LAMAI laboratory, university of Cadi Ayyad, Faculty of sciences and technology, Marrakesh, Morocco

Abstract In this work, a time-fractional anisotropic diffusion equation is employed for image denoising problem. Theoretical results are provided. A discretization scheme by finite difference is also presented. Numerical experiments show a great performance in deleting the noise while preserving important features. Thanks to the fractional derivative robustness and its memory effect potential, the approach delivered remarkable results. Moreover, the model robustness is tested, which is clear visually and quantitatively. All the obtained results conclude that our model surpasses the competitive models, such as Perona-Malik and Weickert.

Low-rank modeling for multidimensional image recovrey

21 Sep.
11h50-12h10

Souad Mohaoui

Abstract Low-rank modeling has achieved great success in various fields including computer vision, and image processing. It refers to a class of methods that solve problems by representing variables of interest using low-rank property. The rank of a multidimensional signal is a precious property which gives an insight into the structural aspects of it. All natural multidimensional signals can be approximated to a low rank signal without losing significant information. For instance,

this low-rank property leads to a proper reconstruction of multi-frame images from a noisy/incomplete version. For various signal processing applications, the underlying low rank nature of signals is widely used. Thus, in this work, we present some applications of low-rank modeling in the context of tensor recovery.

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21 Sep.
12h10-12h30

Image decomposition and denoising by Kantorovich-Rubinstein discrete distance

Omar Oubbih

Laboratoire de Mathématiques Informatiques et Modélisation des Systèmes Complexes, Ecole Supérieure de Technologie d'Essaouira, Université Cadi Ayyad, Maroc

Abstract We propose a new variational model based on the Kantorovich-Rubinstein discrete distance for image decomosition and denoising. We show the theoretical framework and the well-posedness for the proposed model. We apply the first-order primal-dual algorithm of Chambolle-Pock to solve the proposed model. Various experiments show interesting features of the proposed model and yield results which can compare favorably with those obtained by various methods in the literature.

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21 Sep.
12h30-12h50

Fourth-order nonlinear degenerate problem for image decomposition

Ahmed Nokrane

Laboratory LAMAI, Faculty of Science and Technology, University Cadi Ayyad, Marrakech, Morocco

Abstract In this paper, a new coupled fourth order reaction-diffusion system is studied, and applied for image decomposition into cartoon and texture. By Galerkin's method, the existence and uniqueness of an entropy solution to the system with BH initial data are established. Besides, to show the efficiency of the proposed model in image decomposition, numerical experiments and comparisons with other models are done. The acquired results are presented.

Session 2

Global Attractor for a Parabolic Nonlinear PDEs Model Arising in Biological Dynamic in 3D Soil Structure

21 Sep.
15h15-15h35

Mohammed Elghandouri
Faculty of Sciences Semlalia, Cadi Ayyad University, Marrakech, Morocco

Abstract As you know partial differential equations (PDEs) are often used to model, describe, simulate, and understand various natural complex phenomena in many fields. In particular, the fields of biology. It is known also that soil organic matter contains the bulk of carbon on the earth, and we do not deny that there is some works that use PDEs models to simulate microbial decomposition of organic matter but most of them does not take into account the mathematical study of these models. This work is one of the first attempts to study the existence and uniqueness of solution, stability as well as the asymptotic behavior of a PDEs model describing microbial decomposition of organic matter in real 3D soil structure. For this purpose, we develop some arguments and strategies based on semigroup theory to show the existence of such a global attractor of a parabolic nonlinear PDEs model arising in biological dynamic in 3D soil structure. We guarantee to the reader's that these techniques are novel and differ to that can be found in the literature to simulate microbial decomposition of organic matter in soil.

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21 Sep.
15h35-15h55

Pore scale simulation of diffusion using graph neural networks : Application to microbial decomposition of organic matter in soil

Mouad Klai
Faculty of Sciences Semlalia, Cadi Ayyad University, Marrakech, Morocco

Abstract Diffusion occurs continuously in physical, biological, and technological systems, and it is frequently the part of the calculation that demands the most computing power. In the simulation of microbial decomposition in porous media, diffusion is the most computationally expensive part of the calculation. Neural networks trained to provide approximate solutions to such complicated numerical problems, can often provide speed-ups of several orders of magnitude compared to direct calculation. Here we explore a machine learning framework to simulate diffusion at pore scale using a geometrical approach and neural message passing. The neural simulator learns the transport processes in a graph of geometric shapes using numerical techniques derived from Fick's law of diffusion. This work comes after a previous one that was focused on simulating the microscopic scale microbial degradation of organic matter in porous systems. In that work, we used explicit and implicit schemes to model diffusion processes in a graph of geometrical primitives

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21 Sep.
15h55-16h15

Polynomial stability for some partial functional differential equation with distributed delay

Soufiane Boumasmoud
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Abstract Many researchers have been drawn to the study of bilinear systems and control theory throughout the last century. Bilinear systems are a type of nonlinear system that was pioneered in a number of applications such as biological, engineering, social, gas theory, chemical, and physical systems. ([1, 2, 3]) as well as some stochastic processes [4] which can not be modeled through linear approaches.

The use of time delays in mathematical models can enhance the representation of phenomena; yet, the model's complexity may increase dramatically. Furthermore, the presence of delays may result in some impractical instabilities in the system.
In this talk, we study the feedback stabilization of a class of delayed bilinear equation in real Hilbert space. By the help of an observability type inequality and an explicit delayed control we establish sufficient condition ensuring the strong stabilization. Moreover, the speed of convergence is successfully established. Some applications with their numerical simulations are provided.

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21 Sep.
16h15-16h35

Mild solution in the α-norm for some partial integrodifferential equations involving a nonlocal condition

Jaouad ElMatloub
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Abstract In this work, we study the existence of mild solutions for the following partial integrodifferential inclusion:

$$\begin{cases} z'(t) \in Az(t) + \displaystyle\int_0^t B(t-s)z(s)ds + F(t, z(t)) \quad \text{for } t \geq 0 \\ \\ z(0) = z_0 \in X, \end{cases} \tag{1}$$

where $z(\cdot)$ is the state variable taking values in a Banach space $(X, \|\cdot\|)$. The operator A is the infinitesimal generator of a C_0-semigroup $(\mathcal{T}(t))_{t\geq 0}$ on X, $B(t)$ is a closed linear operator with domain $D(B) \supset D(A)$ time-independent, F is a multifunction defined on $\mathbb{R}^+ \times X$.

Our approach adopts the theory of resolvent operator introduced by Grimmer [1] and a fixed point theorem for multivalued condensing maps under hypotheses given in terms of the measure of noncompactness. Finally, an example is given to illustrate the abstract result.

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Session 3

Stability and dichotomy of linear partial differential equations of neutral type

21 Sep.
17h05-17h25

Maryam Lasri
Faculty of Sciences Agadir, University Ibn Zohr, Morocco

Abstract We consider the partial differential equation of neutral type as follow :

$$\begin{cases} \frac{d}{dt}(z(t) - Kz_t) = A(z(t) - Kz_t) + Lz_t, \quad t \geq 0, \\ \lim\limits_{t \mapsto 0}(z(t) - Kz_t) = x \in X, \quad z_0 = \varphi \in L^p([-r, 0], X), \end{cases} \tag{2}$$

where $A : D(A) \subset X \to X$ generates a C_0−semigroup on Banach space X, the operators $L, K : W^{1,p}([-r, 0], X) \to X$ are given by Riemann-Stieltjes integrals and $z_t(\cdot) = z(t + \cdot) : [-r, 0] \to X$.
Actually, we investigated the well-posedness of this equation as well as the property of spectral decomposition of neutral differential equations in infinite dimensional setting, that is the exponential dichotomy. Moreover, we proved that the exponential dichotomy of the associated semigroup to such equations (2) does not depend on that of their associated difference equations: The difference equation

$$\begin{cases} z(t) = Kz_t = \int_{-r}^{0} d\nu(\theta) z(t + \theta), \quad t \geq 0, \\ z_0 = \varphi, \end{cases}$$

with $\nu : [-r, 0] \to \mathcal{L}(X)$ is function of bounded variations continuous and vanish at zero. Based on regular linear systems and feedback theory, we introduced a new transformation of the neutral-type equations

(2) which plays a primordial role in our investigation.

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Boundary null controlability for dynamical parabolic equations

21 Sep.
17h25-17h45

Ghita El Guermai

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Abstract The aim of our paper is to prove the boundary null controllability of the heat equation with dynamic boundary condition, more precisely, we prove that the equation is null controllable at any positive time by means of a boundary control supported on an arbitrary subboundary. The proof of the main result relies on a new boundary Carleman estimate for the adjoint system.

As for the standard (scalar) heat equation, a well-known result states that the boundary null controllability and distributed null controllability are equivalent (see [3, Theorem 2.2]). This result is no longer valid for coupled parabolic systems. In our case, one can obtain the boundary null controllability from a distributed control by enlarging the domain. Nevertheless, this technique for dynamic boundary conditions requires more regular initial data than those in the Dirichlet or Neumann cases (see [27, Theorem 4.5]).
Our boundary Carleman estimate allows us to recover the boundary controllability result without assuming any further regularity assumption.

In comparison with the classical parabolic equations, where the control acts by means of the Dirichlet condition on a part of the boundary of the domain, the present Carleman inequality requires a different technique to absorb a term of Neumann type (in terms of the normal derivative) by the left-hand side of the Carleman estimate. Absorbing this term turns out to be a main difficulty that will be overcome with help of some parabolic regularity estimates. Such regularity estimates can be very useful while doing Carleman estimates [4]. It is worth mentioning the paper [9] which is the first to deal with a Carleman inequality when a time derivative appears in the boundary condition (but without a boundary diffusion). However, it only considers the one-dimensional case and point out the difficulty of the controllability in higher dimensions for such equations (see Section 4 in [9]).

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21 Sep.
17h45-18h05

Internal null controllability for $1-$D heat equation with dynamic boundary conditions

Mariem Jakhoukh
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Abstract The null controllability of parabolic systems has been studied in [1], [2] for Dirichlet and nonlinear Neumann boundary conditions. Recently, Maniar, Meyries and Schnaubelt [3], have considered the case of the dynamic boundary conditions, that tkes the form

$$\partial_t y - \delta \Delta_\Gamma y + \partial_\nu y = 0$$

where the presence of the diffusion term on the boundary i.e "$\delta > 0$", has played an essential role in establishing important results. The case of "$\delta = 0$", i.e. the absence of the diffusion on the boundary, remain unsolved. This situation can occur for instance in the one-dimensional case.

Let $\Omega = (0;1)$ be an open interval of $\mathbb{R}$ and $\omega \subset \Omega$. Consider the following one dimensional heat equation

$$\begin{aligned} \partial_t y - d\partial_{xx} y + a(t,x)y &= 1_\omega(x)v(t,x) && \text{in } (0,T)\times(0,1), \\ \partial_t y(t,j) + (-1)^{j+1}\partial_x y(t,j) + b_j(t)y(t,j) &= 0 && \text{on } (0,T),\ \text{for } j=0,1, \\ (y, y(t,0), y(t,1))_{|t=0} &= (y_0, \beta_0, \beta_1) && \text{in } (0,1), \end{aligned} \tag{3}$$

where the function v is an iternal control. Al. Khoutaibi [4], has established the null controllability of 1D-heat equation (5) using the moment method for real valued control depending only on time t.

In this work, we present the study of both internal null controllability for the 1D-heat equation by developing a Carleman estimate for the adjoint system that will lead to the observability inequality, which is the key to the null controllability. We also present some numerical tests to validate the theoretical results of the null controllability.

keywords: 1D-heat equation, dynamic boundary conditions, Carleman estimate, null controllability, observability inequality.

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Session 4

Extrapolation method for multilinear PageRank computations

22 Sep.
11h00-11h20

Maryam Boubekraoui

FST, University of Cadi Ayyad, Marrakesh, LAMAI, Morocco

Abstract PageRank or Google PageRank is an algorithm that was created for computing a ranking for every web page based on the link structure of the web. The main goal of the PageRank algorithm is computing the principal eigenvector of the Google matrix, which exactly the stationary distribution vector of a Markov chain whose transition matrix is a convex combination of the matrix associated with the web link graph and a certain rank-1 matrix. The PageRank model was extended to the higher order using the higher order Markov processes due to their ability to improve the mathematical modeling and understanding of numerous problems in data and network sciences, such as detecting communities and analyzing spreading dynamics in networks, understanding the behavior of web browsers and drivers trajectories. The solution of this model comes as the principal Z_1-eigenvector of the Google tensor, which is the stationary distribution vector of a higher order Markov chain, and it is called the multilinear PageRank vector. The high-order power method (HOPM) is one of the methods most commonly used to calculate this type of vector, because it is naturally adapted to this problem. However, this method may fail to converge even for irreducible tensors. Moreover, when it converges, its convergence rate can be very slow. In this work, we determine the rate of convergence of this method and improve the convergence by using a vector Aitken-extrapolation method applied to the iterations of the HOPM to obtain an extrapolated approximation of the principal

Z_1-eigenvector.

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Nonlinear boundary periodic equation with arbitrary growth nonlinearity and data measure : Mathematical analysis and Numerical simulation by LBM method

22 Sep.
11h20-11h40

El Ghabi Malika

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Abstract In this work, we are interested in the existence and uniqueness of weak periodic solutions for some quasilinear elliptic equations with data measures and with arbitrary growths with respect to the solution. Since the data are not regular and the growths are arbitrary, a new approach is needed to analyze these types of equations. Since the main operator of our equation is strongly nonlinear, then the classical numerical methods such as finite element, finite difference cannot be used here. for these reasons we propose to use the Lattice Boltzmann method (LBM). We present the complete discretization of this method here. Finally, several numerical examples are given which show the robustness of our algorithm.

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22 Sep.
11h40-12h00

Left invariant Riemannian metrics with harmonic curvature are Ricci-parallel in solvable Lie groups and Lie groups of dimension ≤ 6

Ilyes Aberaouze
Laboratory LAMAI, Faculty of Science and Technology, University Cadi Ayyad, Marrakech, Morocco

Abstract A Riemannian manifold $(M, \langle\ ,\ \rangle)$ has a harmonic curvature if its Ricci tensor Ric is Codazzi, i.e.,

$$\nabla_X(\mathrm{Ric})(Y, Z) = \nabla_Y(\mathrm{Ric})(X, Z) \tag{4}$$

for any vector fields X, Y, Z and where ∇ is the Levi-Civita connection. In particular, any Ricci-parallel metric ($\nabla(\mathrm{Ric}) = 0$) satisfy this condition. To our knowledge, there is no example of a non Ricci-parallel homogeneous Riemannian manifold with harmonic curvature which supports the following conjecture. **conjecture 1:** Any homogeneous Riemannian manifold M with harmonic curvature is Ricci-parallel. This conjecture is true in dimension four (see [7]) and when M is a sphere or a projective space (see [12]). It was proven in [1] for nilpotent Lie groups with left invariant metrics. Moreover, any conformally flat Riemannian manifold with constant scalar curvature satisfies (1) (see [8, Theorem 5.1]) and any homogeneous conformally flat Riemannian manifold has Ricci-parallel curvature (see [13]).

In this talk, we prove that this conjecture is true when the manifold M is a solvable Lie group or a Lie group of dimension ≤ 6.

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Session 5

22 Sep.
12h00-12h20

Flat symplectic Lie algebras

Hamza El Ouali
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Abstract Let (G, Ω) be a symplectic Lie group, i.e, a Lie group endowed with a left invariant symplectic form. If $\mathfrak{g}$ is the Lie algebra of G then we call $(\mathfrak{g}, \omega = \Omega(e))$ a symplectic Lie algebra. The product $\bullet$ on $\mathfrak{g}$ defined by $3\omega(x \bullet y, z) = \omega([x, y], z) + \omega([x, z], y)$ extends to a left invariant connection ∇ on G which is torsion free and symplectic ($\nabla\Omega = 0$). When ∇ has vanishing curvature, we call (G, Ω) a flat symplectic Lie group and $(\mathfrak{g}, \omega)$ a flat symplectic Lie algebra. In this paper, we study flat symplectic Lie groups. We start by showing that the derived ideal of a flat symplectic Lie algebra is degenerate with respect to ω. We show that a flat symplectic Lie group must be nilpotent with degenerate center. This implies that the connection ∇ of a flat symplectic Lie group is always complete. We prove that the double extension process can be applied to characterize all flat symplectic Lie algebras. More precisely, we show that every flat symplectic Lie algebra is obtained by a sequence of double extension of flat symplectic Lie algebras starting from $\{0\}$. As examples in low dimensions, we classify all flat symplectic Lie algebras of dimension ≤ 6.

Solving tensor compressed sensing problem using the Einstein product

22 Sep.
12h20-12h40

Ferdaous Ait Addi
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Abstract Compressed sensing is a new technique that propose to reconstruct a signal from compressed data by exploiting the fact that the most natural signals are sparse in some generic basis. The CS literature has mostly focused on one and two dimensional signals. However, many interesting applications of CS involve higher dimensional signals.

In this work, we propose a new model of higher order compressed sensing using Einstein product. The Einstein product can be seen as a natural extension of the matrix product to higher order arrays, which gives linear algebra tools a natural extension to the higher dimensions. We use greedy algorithms to solve the CS problem.
Key words: compressed sensing, sparse reconstruction, tensors, multilineair decompositions.

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22 Sep.
12h40-13h00

Estimation de certaines sommes courtes

Omar Labihi
FSSM, University of Cadi Ayyad, Marrakesh, Morocco

Abstract Un sujet central en théorie analytique des nombres est l'estimation des sommes de la forme :

$$\sum_{x-y<n\leq x} a_n$$

où (a_n) est une suite de nombres réels ou complexes et $0 < y \leq x$.

L'exposé portera sur des résultats récents concernant ces sommes. Les méthodes utilisées sont d'analyse combinatoire et d'analyse complexe.
(Travail en collaboration avec A. Raouj.)

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Session 6

Performance study of Bernstein–Bézier finite elements for the solution of elastic wave scattering problems in the frequency domain

22 Sep.
15h15-15h35

Nawfel Benatia
LMC, FP Safi, Cadi Ayyad University, Morocco

Abstract Computer simulations of elastic wave propagation and scattering are widely used to tackle a variety of physics and engineering issues, such as traffic vibrations from roads and railways, seismic induced vibrations and other solid structures applications. In medium and high frequency regimes, due to the so-called pollution effect, standard finite elements (FE) require high resolution to maintain engineering accuracy, accordingly resulting in an excessive computational effort making the procedure less efficient.

This work aims at investigating a high-order Bernstein–Bézier FE discretisation to accurately solve time-harmonic elastic wave problems on unstructured triangular mesh grids. Although high-order FEs possess many advantages over standard FEs, the computational cost of matrix assembly is a major issue in high-order computations. A key ingredient to address this drawback is to resort to low complexity procedures in building the local high-order FE matrices. This is achieved in this work by exploiting the tensorial property of Bernstein polynomials and applying the sum factorisation method for curved elements. An efficient implementation of the analytical rules for affine elements is also proposed. Furthermore, element-level static condensation of the interior degrees of freedom is performed to reduce the memory requirements. Additionally, the applicability of the method with a variable polynomial order, based on a simple a priori indicator, is investigated. The performance of the Bernstein–Bézier FEs is

then assessed on various benchmark tests, including the elastic wave scattering and transmission problems.

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22 Sep.
15h35-15h55

On the conditional model in the high dimensional data analysis

Mohamed Alahiane
University of Cadi Ayyad, Marrakesh, Morocco

Abstract We develop new estimation results for the functional relationship between a regressor and a response which are functions indexed by time or by spatial locations. The regressor is assumed to belong to a semi-metric space (E,d) whereas the responses belong to a Hilbert space F. First, we build a double-kernel estimator of the conditional density function, via a NadarayaWatson method. Then, we deduce a conditional mode estimator as the value that maximizes the conditional density estimator. Then, we establish the strong uniform consistencies, with rates, of the two constructed estimators. In this context, we wished to set up these preliminary results which will certainly motivate several works on this same object.

keywords: Derivatives of the conditional density, Conditional mode, Kernel estimation, Functional data analysis.

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On improved shrinkage estimators for modified balanced loss functions

22 Sep.
15h55-16h15

Lahoucine Hobbad
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Abstract This paper considers the problem of estimating the mean vector of $d-$dimensinal spherically symmetric distributed X when the scale parameter is known but when a residual vector U is available: more precisely, let $(X,U) \in \mathbb{R}^{d+k}$ be a random vector around $(\theta,0) \in \mathbb{R}^{d+k}$. The loss functions is assumed to be modifications of balanced loss functions the form:
(i) $\omega\rho(\|\delta-\delta_0\|^2)+(1-\omega)\rho(\|\delta-\theta\|^2)$ and **(ii)** $\ell(\omega\|\delta-\delta_0\|^2+(1-\omega)\|\delta-\theta\|^2)$,
where δ_0 is a target estimation of $\gamma(\theta)$, and where ρ and ℓ are increasing and concave functions. For $d \geq 4$ and the target estimator $\delta_0(X)=X$, we provide the estimators of the form $\delta_{\omega,g}(X)=X+a\|U\|^i(1-\omega)g(X)(i=0,2)$ dominate $\delta_0(X)=X$ and are minimax where we suppose there exists a nonpositive function $h(.)$ such that $h(X)$ is subharmonic and $E_{R,\theta}\left[R^2h(W)\right]$ is nonincreasing with $W \sim U_{R,\theta}$, $E_\theta\left[|h(X)|\right]<\infty$ and such that $g(X)$ is weakly differentiable and also satisfies :

(i) $div(g(X)) \leq h(X)$, **(ii)** $\|g(X)\|^2 + 2h(X) \leq 0$.

Keyword : Modified balanced loss, Baranchik type estimators, Dominance, Minimax estimator, Robust shrinkage estimation, spherically symmetric distribution with residual.

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Session 7

Impulse controllability for degenerate singular parabolic equation

22 Sep.
17h05-17h25

Ilham Ouelddris
FSSM Cadi Ayyad University

Abstract Let ω be a nonempty subinterval of $(0,1)$. For $T>0$, we denote $Q=(0,1)\times(0,T)$ and we consider the following impulse singular degenrate system

$$\begin{cases} \partial_t y-(x^\alpha y_x)_x-\dfrac{\mu}{x^\beta}y=0, & \text{in } (0,1)\times(0,T)\backslash\{\tau\}, \\ y(\cdot,\tau)=y\left(\cdot,\tau^-\right)+\mathbb{1}_\omega h, & \text{in } (0,1), \\ y(1,t)=0, & \\ \begin{cases} y(0,t)=0, & (WD), \\ (x^\alpha y_x)(0,t)=0, & (SD), \end{cases} & \text{on } (0,T), \\ y(0,x)=y_0(x), & \text{on } (0,1), \end{cases} \tag{5}$$

where $y_0\in L^2(0,1)$, $y(.,\tau^-)$ is the left limit of y at time $\tau\in(0,T)$ and $\mathbb{1}_\omega$ is the characteristic function of ω. Next, we consider the non-impulsive singular degenerate system

$$\begin{cases} \partial_t u-(x^\alpha u_x)_x-\dfrac{\mu}{x^\beta}u=0, & \text{in } Q, \\ u(1,t)=0, & \\ \begin{cases} u(0,t)=0, & (WD), \\ (x^\alpha u_x)(0,t)=0, & (SD), \end{cases} & \text{on } (0,T), \\ u(0,x)=u_0(x), & \text{on } (0,1). \end{cases} \tag{6}$$

The parameters α, β and μ are supposed to correspond to one of the three assumptions Sub-critical potentials:

$$\alpha \in [0,2[\,, \quad 0 < \beta < 2-\alpha \quad and \quad \mu \in \mathbb{R}; \tag{7}$$

$$\alpha \in [0,2[\,\backslash\{1\}, \quad \beta = 2-\alpha \quad and \quad \mu < \mu(\alpha); \tag{8}$$

critical potentials:

$$\alpha \in [0,2[\,\backslash\{1\}, \quad \beta = 2-\alpha \quad and \quad \mu = \mu(\alpha); \tag{9}$$

where $\mu(\alpha) = \dfrac{(1-\alpha)^2}{4}$ is the optimal constant of the classical Hardy-Poincaré inequality.

The aim of this work is to prove a logarithmic convexity estimate of the singular degenerate equation 6 by using the Carleman commutator approach traducing an observability inequality at one instant of time. As an application, we deduce the impluse null approximate contrallability of the system 5.

keywords: Singular degenarate equation ,The impulse null approximate controllability, logarithm convexity estimate, observability inequality.

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22 Sep.
17h25-17h45

Null controllability for stochastic parabolic equations with dynamic boundary conditions.

Abdellatif Elgrou
FSSM Université Cadi Ayyad

Abstract In this work, we establish the null controllability for general forward linear stochastic parabolic equations with dynamic boundary conditions. The main tool will be the global Carleman estimates for this type of boundary condition called also of Wentzell type.

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Uniform Boundary Controllability of a Semidiscrete 1-D Coupled Wave Equation with Vanishing Viscosity

22 Sep.
17h45-18h08

Sara Muntassir
FSSM Cadi Ayaad University

Abstract We study the controllability of a semi-discrete system obtained by discretizing in space the coupled 1-D wave equation with a boundary control at one extreme. The finite difference and finite element introduce high frequency spurious oscillations that lead to non-uniform controllability as the discretization parameter goes to zero (see [1]). To filter these high numerical frequencies we add numerical vanishing term to each equation, which ensures the convergence of the sequence of discrete controls to a control of the continuous coupled wave equation when the mesh size tends to zero.

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Session 8

Bohr-Neugebauer type result for a class of nondensely nonautonomous linear partial functional differential equations.

23 Sep.
11h00-11h20

Afoukal Abdallah
FSSM Université Cadi Ayyad

Abstract The aim of this talk is to extend the Bohr-Neugebaur type theorem to a class of non-autonomous linear partial functional differential equations with Stepanov almost periodic forcing term. Our approach is based on a variation of constants formula and Bohr-Neugebaur type theorem for linear difference equations. To illustrate our main results, we propose an application to a reaction-diffusion equation with continuous delay.

New principle reduction for partial functional differential equation with the lack of compactness.

23 Sep.
11h20-11h40

El Attaouy Meryem
FSSM Université Cadi Ayyad

Abstract In this talk we are interested to investigate the existence of almost automorphic and almost periodic solutions for some partial functional differential equation with the lack of compactness of the semigroup generated by the linear part. Under condition more general than the compacity of the C_0-semigroup generated by the linear part we establish the quasi-compactity of the C_0-semigroup associated to the linear part of the partial functional differential equation. This property allows as to construct a reduced system that is posed in a

finite dimensional space. Through this result we investigate the existence of almost automorphic and almost periodic solutions for some partial functional differential equations. We apply our results to transportation models.

Keywords: Functional differential equations; Quasi-compact semigroup; Variation of constants formula; Stepanov-almost automorphic solution.

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23 Sep.
11h40-12h

The heavy ball method regularized by Tikhonov term. Simultaneous convergence of values and trajectories.

Akram Chahid Bagy
FSSM Université Cadi Ayyad

Abstract In a real Hilbert space $\mathcal{H}$, let $f : \mathcal{H} \to \mathbb{R}$ be a convex differentiable function whose solution set $\operatorname{argmin}_{\mathcal{H}} f$ is nonempty. To attain a solution of the problem $\min_{\mathcal{H}} f$, we consider the second order dynamic system

$$\ddot{x}(t) + \alpha\, \dot{x}(t) + \beta(t)\nabla f(x(t)) + cx(t) = 0,$$

where $\beta(.)$ is a $\mathcal{C}^2$ positive nondecreasing function such that $\dot{\beta}(t) \neq 0$ for t large enough, with

$$\lim_{t\to+\infty} \beta(t) = +\infty \quad ; \quad \lim_{t\to+\infty} \frac{\dot{\beta}(t)}{\underline{(}t)} = 0 \quad \text{and} \quad \limsup_{t\to+\infty} \frac{-\ddot{\beta}(t)}{\dot{\beta}(t)} < \frac{\alpha}{2},$$

and α, c are two positive constants with $c \geq \alpha^2$. Let us introduce the real function $\phi_t : \mathcal{H} \longrightarrow \mathbb{R}$ defined by

$$\phi_t(x) := f(x) + \frac{c}{2\beta(t)}\|x\|^2.$$

Note that ϕ_t is $\frac{c}{\beta(t)}$-strongly convex, and set, for each $t \geq t_0$

$$x_{\beta(t)} := \operatorname*{argmin}_{\mathcal{H}} \phi_t$$

which is the unique minimizer of the strongly convex function ϕ_t.
The following properties are immediate consequences of the classical properties of the Tikhonov regularization

$$\forall t \geq t_0 : \quad \|x_{\beta(t)}\| \leq \|x^*\|,$$

$$\lim_{t\longrightarrow+\infty}\|x_{\beta(t)}-x^*\|=0 \quad \textit{where } x^*=\text{proj}_{\underset{\mathcal{H}}{\text{argmin}}\, f}(0).$$

We prove that the value of the objective function in a generated trajectory converges in order $\mathcal{O}\big(\frac{1}{\beta(t)}\big)$ to the global minimum of the objective function, that the trajectory strongly converges to the minimum norm element of $\text{argmin}_{\mathcal{H}} f$ and that $\|\dot{x}(t)\|$ converges to zero in order $\mathcal{O}\big(\sqrt{\frac{\dot{\beta}(t)}{\beta(t)}}+e^{-\mu t}\big)$ where $\mu<\frac{\alpha}{2}$.
We then present two choices of β to illustrate these results, the first is $\underline{(}t)=t^p\ln^q(t)$ (with $(p,q)\in(\mathbb{R}_+)^2\setminus\{(0,0)\}$), we obtain in this case, as $t\to+\infty$, the following convergence rates:

i) $\quad f(x(t))-\underset{\mathcal{H}}{\min} f=\mathcal{O}\left(\frac{1}{t^p(\ln(t))^q}\right);$

ii) $\quad \|x(t)-x_{\beta(t)}\|^2=\begin{cases}\mathcal{O}\left(\frac{1}{t\ln(t)}\right) & \textit{if } \ p=0\\ \mathcal{O}\left(\frac{1}{t}\right) & \textit{otherwise} \ ;\end{cases}$

iii) $\quad \|\dot{x}(t)\|^2=\begin{cases}\mathcal{O}\left(\frac{1}{t\ln(t)}\right) & \textit{if } \ p=0\\ \mathcal{O}\left(\frac{1}{t}\right) & \textit{otherwise} \ .\end{cases}$

The second is $\beta(t)=e^{\gamma t^p}$ (with $p\in]0,1[$ and $\gamma>0$), in this case, we get as $t\to+\infty$, the following convergence rates:

$$f(x(t))-\underset{\mathcal{H}}{\min} f=\mathcal{O}\left(e^{-\gamma t^p}\right),$$
$$\|x(t)-x_{\beta(t)}\|^2=\mathcal{O}\left(\frac{1}{t^{1-p}}\right)$$
$$\text{and} \quad \|\dot{x}(t)\|^2=\mathcal{O}\left(\frac{1}{t^{1-p}}\right).$$

Finally, On the basis of the Moreau regularization technique, we extend these results to non-smooth convex functions with extended real values.

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Session 9

23 Sep.
12h00-12h20

Nonlinear degenerate parabolic quasi-variational inequalities

Sana Ajagjall
ESTE, University Cadi Ayyad, Marrakesh

Abstract This work is devoted to study the existence of solutions for some nonlinear degenerate quasi-variational inequalities with homogenous Dirichlet boundary conditions. We investigate the existence of solution in different cases with respect to the various regularity of a given data. We also study the asymptotic behaviour as t tends to infinity and we identify the limiting problem.

keywords:
Quasi-variational inequalities, Asymptotic behaviour in time, Gradient constraint, Degenerate problem.

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23 Sep.
12h20-12h40

Darbo's fixed point under weak topology features with application to a functional integral equation

Mohamed Khazou
Laboratory of Complex Systems Modeling, National School of Applied Sciences, University Cadi Ayyad, Marrakesh

Abstract This talk is dedicated to generalizing some of Darbo's type fixed point theorems by weaking the set contractive condition, a methodology is based on the concept of operators $O(\bullet;.)$, and JS-contractive condition in Banach spaces. Furthermore, some new coupled fixed point results will be given.

The measure of weak noncompactness is the main tool in the presentation of our proofs, without requiring weak compactness or weak continuity. Further, we apply the obtained results to prove the existence of solutions for a Hammerstein integral equation.

Key words : Fixed point theorem, Coupled fixed point, Measure of weak noncompactness, Weak topology, Weakly sequentially continuous operator, Ws-compact, Hammerstein integral equations.

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23 Sep.
12h40-13h00

Random fixed point theorems with application to random differential equations in Banach spaces

Adil El Ghabi
Cadi Ayyad University, National School of Applied Sciences
Marrakesh, Morocco

Abstract In this talk, we present several random fixed point theorems for (countably) convex-power condensing random operators in Banach spaces. We also present some new random fixed point results for monotone (countably) convex-power condensing random operators in ordered Banach spaces. As an application, we discuss the solvability of a broad class of random first-order vector-valued ordinary differential equations.

Keywords: Random (deterministic) fixed point, random operator, weak topology, measure of (weak) noncompactness, (countably) convex-power condensing, monotone operator, random differential equation.

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Session 10

Diameter, upper traceable numbers and radio k-labelings of circulant graphs

23 Sep. 15h45-16h05

Laila Loudiki
Polydisciplinary faculty of Safi,Cadi Ayyad university, Morocco

Abstract Motivated by problems in radio channel assignments, we consider radio k-labelings of graphs [1]. Let G be a connected graph. For any positive integer $k \geq 1$, a radio k-labeling f of G is an assignment of non-negative integers to the vertices of G such that

$$|f(u) - f(v)| \geq 1 + k - d_G(u, v),$$

for every two distinct vertices u, v of G, where $d_G(u, v)$ is the distance between u and v in G. The span $rn_k(f)$ of a radio k-labeling f of G is the maximum color (positive integer) assigned to a vertex of G. The minimum span of all radio k-labelings of G is called the radio k-labeling number $rn_k(G)$ of G.

In this paper, we study radio k-labelings of circulant graphs $C_n(D)$, i.e., graphs with the set $\{0, 1, \ldots, n-1\}$ of integers as vertex set and in which two distinct vertices $i, j \in \{0, 1, \ldots, n-1\}$ are adjacent if and only if $|i - j|_n \in D$, where $|x|_n = \min(|x|, n - |x|)$ is the cyclic absolute value of an $x \in \mathbb{Z}$. We shall also study the upper traceable number [2] of $C_n(D)$. This graph parameter is denoted by

$$t^+(G) = \max_{\sigma} \sum_{i=0}^{n-2} d_G(\sigma(i+1), \sigma(i)),$$

the maximum being taken from all the permutations σ of the vertices of G.

We give exact values for the diameter, the upper traceable numbers as well as the radio k-labeling numbers of circulant graphs $C_n(D)$.

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23 Sep.
16h05-16h25

The Omega-spectrum of polynomial functions

Zakaria Taki

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23 Sep.
16h25-16h45

A General Construction of Spherical Barycentric Coordinates and Applications

Alaa eddine Bensad
Faculty of Science and Technology, Cadi Ayyad University (UCA), Marrakesh, Morocco

Abstract We provide a general method for defining and efficiently computing barycentric coordinates with respect to polygons on the unit sphere. More precisely, we develop a novel explicit construction which allows us to compute the spherical barycentric coordinates from their 2D-Euclidean counterparts. In particular, we give two interesting families of spherical coordinates, one is defined for convex and non-convex spherical polygons. An interesting consequence is the possibility to construct new 3D barycentric coordinates for arbitrary polygonal meshes. Furthermore, we present an alternative construction for spherical barycentric coordinates with help of 3D barycentric coordinates for closed triangular meshes. This construction is

extended to arbitrary dimensions. We show that our spherical and 3D coordinates are widely applicable to many domains. We give several examples related to spherical blending, space deformations and shape morphing in 3D.
Keyword :
Geodesic Polygon, Spherical Barycentric Coordinates, 3D Barycentric Coordinates, Morphing.

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